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On Nonlinear Viscoelasticity of Elastomeric ISOLGOMMA Mats

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Abstract: This paper is devoted to an analysis of experimental results on elastomeric mats applied to the pavement on the underground station "Belgrade-junction" ("Beograd-čvor" in Serbian). The elastomeric composite mat taken as a support of the underground tracks is called ISOLGOMMA-M40AV. For an appropriate modelling a nonlinear viscoelasticity model is needed. The Rabotnov's nonlinear hereditary integral equation with Abelian kernel is applied and calibrated from triangular history stress controlled testing results.

Keywords: High velocity trains, elastomeric damping, viscoelastic dissipation.

1 Introductory remarks on thermo-mechanical features of elastomers

The subject of our interest is the fact that the elastomeric composite named ISOLGOMMA-M40AV is applied as a damping support at the station "Belgrade-junction" ("Beograd-čvor" in Serbian). We are especially interested in a mechanical modeling of such a material. Due to this goal, let us give first a small introduction on elastomers and then to concentrate our attention to this specific material and available experimental data trying to explain them analytically.

In the environment where fast changing cyclic loadings and temperature fields take place an application of elastomeric materials leads to considerable increase of life time, abrasive durability and reduction of noise and vibration if compared with rigid structures.

Determination of their thermoviscoelastic properties has been an everlasting problem directed to improvement of the mentioned strength and life time.

Elastomers could be defined as polymers or polymer based composites which in the whole range of exploitation exhibit ability to sustain very large (even of the order of strain equals to 100%) viscoelastic reversible strains.

A technical rubber is a highly elastic material obtained by the vulcanization which is a chemical process for converting natural rubber or related polymers into more durable materials via the addition of sulfur or other equivalent additives (eg. peroxide-based systems).

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The natural rubber is sticky, deforms easily when warm, and is brittle when cold. This lack of elasticity is very inconvenient when a structure is foreseen to stand diverse temperatures ranges. The reason for inelastic deformation of un-vulcanized rubber can be found in its chemical structure: rubber is composed of long polymer chains. These chains can move independently relative to each other, which lets the material change shape. Crosslinking introduced by vulcanization prevents the polymer chains from moving independently. As a result, when stress is applied to a vulcanized rubber body it deforms, but upon release of the stress, the article reverts to its original shape.

There exist at least two methods for vulcanization. The economically most important method (compression molding) uses high pressure and temperature. The rubber article is intended to adopt the shape of the mold. In the second method either a hot air vulcanization or a microwave heated vulcanization (both continuous processes) is applied.

The elastomer named ISOLGOMMA ("rubber mat" or "MEGAMAT" being actual name of this material) is aimed to be applied as a support for trains and tramway vehicles reducing noise and vibration [1, 2]. The Italian enterprise having the same name has patented this material composed of recycled car tire rubber and some granules connected by a special polyurethane binder. This gives high durability.

Without entering too much into chemical procedures the producer claims that the product has the following *basic features of Isolgomma:*

(a) low modulus of elasticity,
(b) high damping,
(c) excellent dimensional stability,
(d) almost steady performance under mechanical loadings and aging and
(e) low sensibility to temperature variations and chemical agencies.

Finally, it should be mentioned that this enterprise is just one of great many existing in the world. For instance, a Russian experience in ship dampers as presented in [3] is also very interesting and important.

2 A low cycle fatigue testing by pressure of an ISOLGOMMA specimen

A static testing necessarily includes slow temporal loading change according to rules defined in advance. The method is called *stress controlled testing*. In our case such a testing has been performed by a electromechanical uniaxial testing machine SCHENCK TREBEL. A uniaxial static compressive loading onto a parallelepiped specimen with dimensions $300 \times 300 \times 38\ mm^3$ was made according to the UNI 10570 standard. The producer had given program conditions with three loading-unloading cycles foreseen. Each phase either of loading or unloading was 1 minute long. The triangular history of loading is depicted in the following figure.

Recording of the three stress-strain curves was done. This curves are convenient for a modeling which is given in the next section.

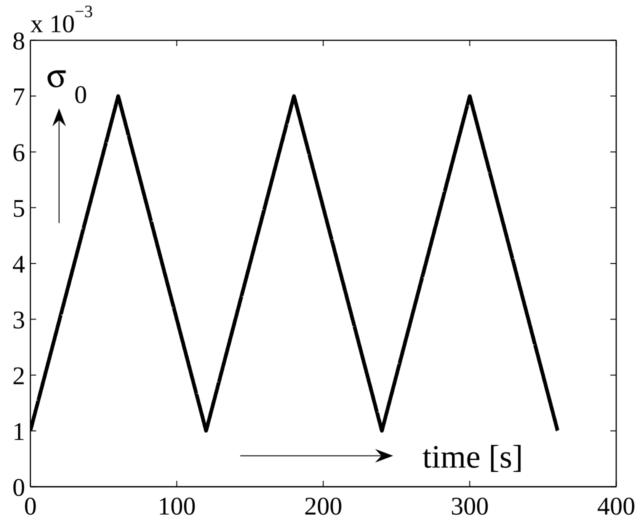

Fig. 1. The triangular loading static history applied to parallelepiped specimen.

3 Modeling of vibro-creep of the damaged elastomeric composite ISOLGOMMA

An analysis of the obtained testing data is much more accurate if we instead of a linear viscoelastic model choose a nonlinear viscoelasticity constitutive equation as in the paper [4]. Let us take our approach to Rabotnov memory model [5] with a hereditary integral equation:

$$\varphi\big(\varepsilon(t)\big) = \sigma(t) + \int_0^t R(t-\xi)\sigma(\xi)d\xi. \tag{3.1}$$

For practical reasons of easier calculations the kernel R is chosen either as Abelian or degenerate. With first choice we have [5]:

$$R(\xi) = \tau_0\xi^{-\alpha}/\Gamma(1-\alpha) \equiv \alpha_1\xi^{\alpha_2},$$

where the nonlinear function φ is commonly taken as a low order polynomial:

$$\varphi(\varepsilon) = \sum_{k=1}^{M} C_k\,\varepsilon^k.$$

For $M = 3$ and three low frequency "quasistatic" triangular stress history (with the excitation frequency $\Omega = 2\pi/120 \approx 0.05[s^{-1}]$) material constants of the chosen model are determined by some numerical routine (like in MATLAB) and presented in the enclosed table 1.

Quality of the modeling is illustrated by the figure 2 where continuous line is chosen for analytical prediction whereas small cyrcles mean experimental data.

Obviously, the precision is high since the analytical model and test data almost coincide. However, an interpretation of damage is much more shadowed than in the case of linear

Table 1. Material constants $\{\alpha_{1,2}\ \&\ C_{1,2,3}\}$ for the three cycles of Rabotnov model with Abelian kernel (η is the corelation coefficient).

Cycle	α_1	$log(\alpha_2)$	$\{C_1, C_2, C_3\}$	η
First	0.474	-39	$\{1.15\times10^{-3}, -3.11\times10^{-6}, -7.32\times10^{-9}\}$	0.99
Second	0.751	-155.8	$\{-1.1\times10^{-3}, 3.9\times10^{-6}, -3.1\times10^{-9}\}$	0.993
Third	0.138	-162.6	$\{-3.2\times10^{-3}, 8.8\times10^{-5}, -4.7\times10^{-7}\}$	0.992

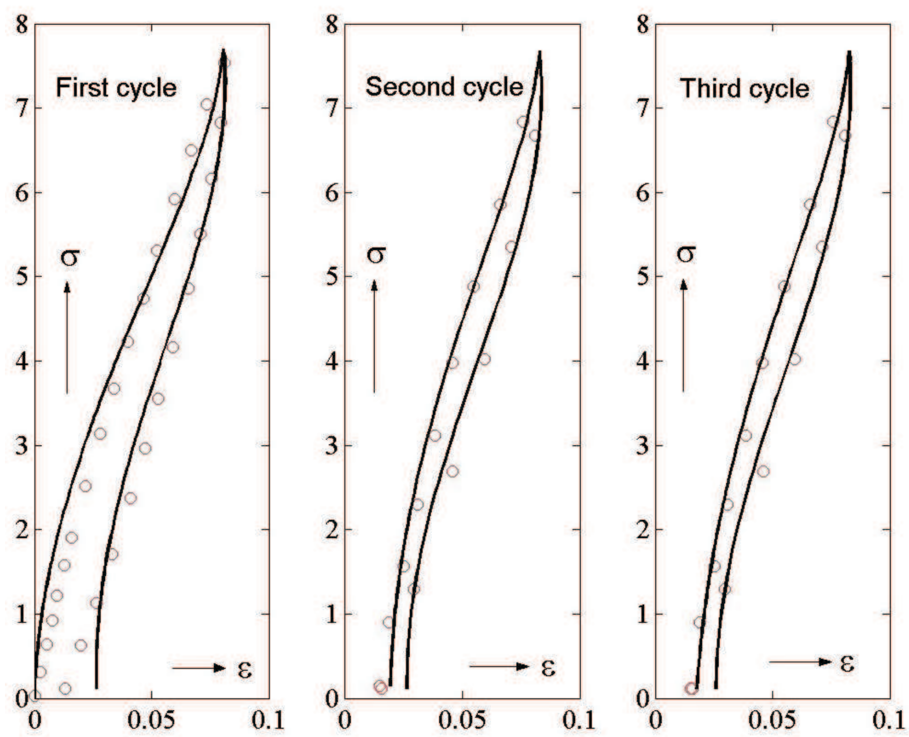

Fig. 2. Evolution of the low frequency creep for ISOLGOMMA mat modelled by the nonlinear viscoelastic Rabotnov's integral equation

model. An attempt in the paper [4] has been made where it was suggested to replace in (3.1) the true stress by some effective stress including a scalar measure of damage. Such a measure is necessarily scalar since it was assumed for the second rank damage tensor to be spherical without deviatoric part. An evolution equation for damage variable like in [6] could be taken. Such an equation is far from being perfect and represents the crudest approximation.

4 Concluding remarks

We may conclude that

◇ Dynamic high frequency tests (being most interesting from the noise standpoint) could be approximately modeled by the linear viscoelasticity. However a more precise analysis requires a nonlinear viscoelasticity approach as mentioned in [2].

◇ Damage evolution may be modeled and understood by the self consistent method (cf. [8–10, 12]) with a help of the involved thermomechanical analysis of residual stresses (i.e. eigenstresses) and fracture mechanics methods for pore development with inclusion geometry described as in [11]. The trouble is, however, in the fact that the inevitable linearity of this method does not correspond to the experimental data. Some nonlinear variational approach would be promising in the future work.

◇ The nonlinear viscoelasticity briefly presented here covers very well the lowfrequency quasistatic tests with triangular excitation history. Anyway, a delicate question concerning a well defined damage interpretation remains unsolved.

◇ Some new experiments with impact stress loading characteristic for Hopkinson bar devices would be precious for high frequency processes being expected especially at high velocity trains.

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References

[1] *Elaboration no. 2-257-99 on examination of the elastomer Isolgomma tip M40A*, Beogradčvor, Beograd, 1999.

[2] M. SANTINI, *Prodotti per il controllo delle vibrazioni. Determinazione delle carratteristiche meccaniche di materassini e piastre*, UNI 10570, Bozza n.1 - Maggio 92., Milano, 1992.

[3] A.V. IONOV, L.E. BUVAILO, M.V. VOLKOVA, A.P. STAROSTIN, *Elastomer materials in ship vibration and noise protection means*, Russian Journal of General Chemistry, , Vol. 53, No. 4, (2010), 41-–53.

[4] LJ. KUDRJAVCEVA, D. ŠUMARAC, M. MIĆUNOVIĆ, *On inelastic properties of continuously damaged elastomers*, Proceedings of the Yugoslav Congress of Mechanics, 2003, Beograd

[5] YU. N. RABOTNOV, *Elements of hereditary solid mechanics*, Mir Publishers, Moscow, 1980

[6] G. DRAGANOVA, K. MARKOV, *On a model of damage in polymer materials*, Proceedings Continuum Models and Discrete Systems 8., (K. Markov ed.) , World Scientific, Singapore,1996, pp. 615-622

[7] D. Krajčinović, *Damage mechanics*, Elesevier, Amsterdam, 1996.

[8] I. A. Kunin, *Elastic Media with Microstructure*, Springer Series in Solid State Sciences, Berlin, 1983.

[9] M. Mićunović, Lj. Kudrjavceva, D. Šumarac, *Thermo-inelasticity of Damaged Elastomers by Self Consistent Method*, Int. J. Damage Mechanics, Vol. 17, No. 6 (2008), 539-565.

[10] M. Mićunović, *Self-consistent method applied to quasi-rate dependent polycrystals* Philosophical Magazine, Vol. 85 (33-35), (2005), 4031–4054.

[11] M. Mićunović, Lj. Kudrjavceva, D. Šumarac, *Thermoelasticity of damaged elastomers - symmetry issues* J. Phys. Conf. Ser., Vol. 62, (2007), 88-101.

[12] D. Šumarac, D. Krajčinović, *A self-consistent model for microcrack weakened solids*, Mech. Materials, Vol. 6, (1987), 39.