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The Effect of a Change in the Shape of a Solar Sail due to Electrization on the Feasibility of an Interplanetary Flight along the Tsander Trajectory

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Abstract: The study of the motion of spacecraft with a solar sail is related to the projects that have been actively developed recently for flights to the borders of the solar system and their use for maneuvering in the vicinity of libration points. When planning and implementing these projects, it is necessary to take into account the change in the properties of the sail surface during flight. In particular, as shown earlier, the electric charge induced on the surface of the sail leads to deformation of its surface. The present work is devoted to the study of the effect of deformation of the sail surface on its dynamics. For certainty, the movement along the Tsander trajectory of the flight from Earth's orbit to Mars' orbit is chosen. It is shown that even for small relative deformations of the sail surface, the trajectory deviation from the calculated one can exceed the radius of Mars. The results obtained make it possible to determine the optimal parameters of the transverse stiffeners of the solar sail for the realization of permissible relative deformations in the Earth's orbit.

Keywords and phrases: cosmic plasma, electric charge, shape changing, solar sail spacecraft, surface electrization, Tsander trajectory

1 Introduction

The development of space exploration required the development of fundamentally new propulsion systems capable of operating with minimal consumption of onboard energy sources. Such systems include, for example, various sailing structures: conventional solar sail [25, 27, 20, 1], electric solar sail [9, 24, 8, 13], dipole solar sail [33], magnetic solar sail [31, 2, 32, 4] and other more extravagant systems [5, 17, 21].

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An electric solar sail consists of a series of long and thin cables with a high positive potential, which are deployed when the spacecraft is rotated around its axis of symmetry. The positive charge of the cables is created by injecting a beam of electrons directed against the motion of the spacecraft. Positively charged cables repel the ions of the solar wind, which leads to the transfer of momentum from the ions to the sail and, consequently, to the acceleration of the spacecraft.

The thrust of a magnetic solar sail arises as a result of the magnetostatic interaction between the solar wind and an artificial magnetic field, which is produced on board by an electric current flowing through an electrical circuit made of conductive material.

Despite the detailed development of the concepts of electric and magnetic solar sails and conducting field tests of an electric solar sail [13], the concept of a conventional film solar sail is the most discussed and practically implemented [15].

Solar sails were first proposed by K.E. Tsiolkovsky and F.A. Tsander back in the 1920s. The sail is a large sheet of low-density material covered with a reflective layer, the only source of energy of which is the electromagnetic flux of the Sun. At the same time, the problem of deploying and maintaining the shape of the solar sail, which is a huge membrane, is urgent. The known solutions to this problem are based either on the use of stiffeners or on the use of the rotating solar sail concept [7].

In theory, flying with a solar sail can last indefinitely, due to the "constant gentle exposure to sunlight". Interest in solar sails increased in the 1970s due to theoretical research by NASA, although no practical missions using them were conducted at that time. In the following decades, many theoretical and experimental papers on solar sailing were published.

In addition to studying the dynamics of interplanetary flights, the use of a solar sail for maneuvering in the vicinity of certain points of particular interest in outer space, such as libration points, has recently been actively studied [6, 18, 26, 16, 19].

A separate important area of research into the motion of spacecraft with a solar sail is the study of the influence of various space factors on their dynamics, such as film aging [10, 11, 3] and its electrification under the influence of solar radiation and cosmic plasma [29, 22].

In paper [30], a method was proposed for calculating the surface charge density of a flat three-layer plate located in a cosmic plasma, one of the sides of which is also illuminated by a stream of electromagnetic radiation from the Sun. The plate is a layer of dielectric coated on both sides with a thin conductive (aluminum) layer. This formulation of the problem corresponds to the motion of a spacecraft with a solar sail along the Tsander trajectory when the sail plane is oriented normally to the Sun's rays. The problem of electrization a three-layer plate was solved under the following assumptions:

- equality of electrostatic potentials of both conducting surfaces;

- the existence of a double layer with a non-monotonic potential distribution over the illuminated side of the plate;

- the size of the sail is much larger than the Debye radius of the surrounding plasma;

- the speed of a spacecraft with a sail is much less than the speed of the solar wind;

- when calculating the surface charge density, the influence of the sail boundaries is not taken into account;

- the distribution functions of charged particles at infinity are considered to be known.

The obtained results of charge densities for different plate distances from the Sun allowed us to formulate a number of problems on the effect of induced charge on the dynamics of spacecraft with a solar sail. In particular, it was shown in [30] that an electric charge induced on the surface of a sail that does not change shape does not significantly affect the dynamics of a spacecraft during an Earth-Mars flight along the Tsander trajectory due to the smallness of the interplanetary magnetic field.

At the same time, the surface charge on a solar sail is capable of changing the shape and area of the sail's midsail, which must be taken into account when controlling the motion of a spacecraft with a sail [22].

A similar problem of determining the natural oscillation shapes of a three-layer rectangular plate modeling a solar sail for the OKEANOS project was considered in [12]. In this paper, the effect of changing the shape of the sail on the dynamics of a spacecraft with a sail when moving along the Tsander trajectory is investigated. Two flight options are being considered. The first flight is on the Earth-Mars route, and the second one is on the Venus-Earth route.

2 Deformation of a solar sail under the influence of an electric change on its surface

In [22], a finite element model of a solar sail developed in the ANSYS software package was described and its deformation was calculated for previously determined electric charge densities on its surface. The solar sail was approximated by a round three-layer plate, the inner layer of which is a mylar film of thickness d, covered on both sides with a very thin aluminum film one tenth of a thickness d. An electric charge with densities σ_1 and σ_2, respectively, is evenly distributed on the upper and lower sides of the plate. It was assumed that under the action of a surface force, the plate in question deforms as part of the surface of the sphere, and the following expression is valid for the surface force [23].

$$T = 2\pi k_0 \sigma_i^2 R, \quad (i = 1,2) \tag{1}$$

where $k_0 = 9 \cdot 10^9 N \cdot m^2 / Kl^2$, and R is the radius of curvature of the surface (in meters). During the approximation, the area occupied by the plate was divided into more than 10,000 elements. The program used a three-layer, eight-node "shell91" element designed for modeling thin layered shells. Article [22] presents the calculation results for sails with radii up to 50 m and $d = 2$ microns. Fig. 1 shows the result of computer simulation of the solar sail deformation.

Fig. 2 shows the dependence of the relative deformation $\varepsilon = \Delta R / R_s$ on the radius of the sail R_s in Earth's orbit, where ΔR is the deflection of the sail. It can be seen that as the plate radius increases, the value of ε increases. It is shown that starting from a radius of 50 meters, the plate does not keep its shape, dents appear on it and, consequently, there is

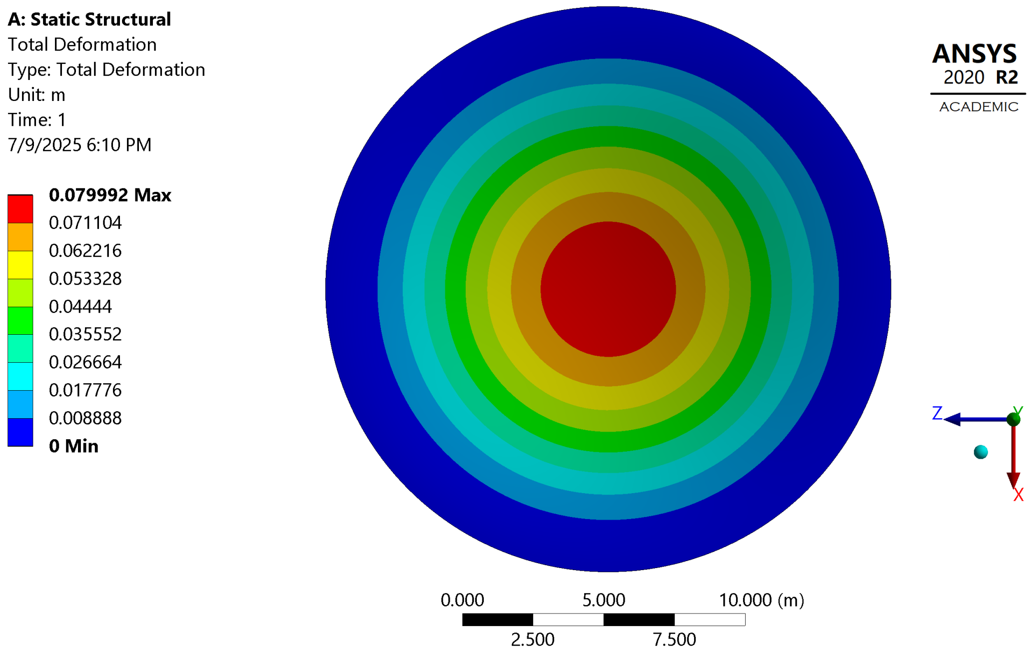

Figure 1. Solar sail deformation.

no uniform radius of curvature. Thus, for a flight along the Earth-Mars route, transverse stiffeners are needed to maintain the shape of the plate [22].

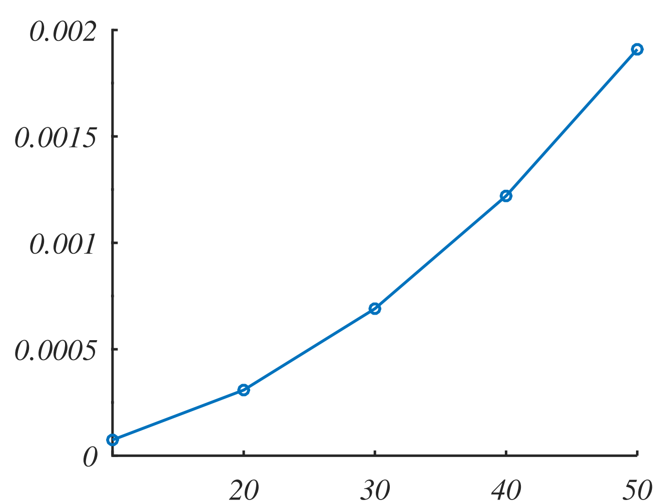

Figure 2. Relative deformation of the sail versus its radius (in meters).

3 Determination of solar radiation pressure on a deformed shell

Suppose that, given a sufficient number of stiffeners, a solar sail deformed by surface force can be represented as a spherical segment of radius R with a solution angle of 2α (Fig. 3).

In this case, the relative deformation can be expressed as follows

$$\varepsilon = \frac{\Delta R}{R_s} = \frac{R(1-\cos\alpha)}{R\alpha} = \frac{(1-\cos\alpha)}{\alpha}.$$

Thus, the angle α is the solution of the following equation

$$\varepsilon\alpha = 1-\cos\alpha. \tag{2}$$

Summarizing the results of [28] for the case of a spherical segment, we obtain formulas for the force of solar radiation pressure on the solar sail model under consideration:

$$F = \frac{2\pi R_s^2 I_0}{c_0}(q_1+q_2), \tag{3}$$

where

$$q_1 = \frac{1}{\alpha^2}\int_0^{\alpha} \sin\theta\cos\theta[1+\cos 2\theta]\langle R_p\rangle\, d\theta, \tag{4}$$

$$q_2 = \frac{1}{\alpha^2}\int_0^{\alpha} \sin\theta\cos\theta[1+\cos(\theta-\theta_2)]\langle T_p\rangle\, d\theta, \tag{5}$$

$$\langle R_p\rangle = \frac{1}{2}\left[\frac{\tan^2(\theta-\theta_2)}{\tan^2(\theta+\theta_2)} + \frac{\sin^2(\theta-\theta_2)}{\sin^2(\theta+\theta_2)}\right],$$

$$\langle T_p\rangle = 1-\langle R_p\rangle,$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta}{n_2}\right), \tag{6}$$

I_0 is the intensity of incident radiation, c_0 is the speed of light in a vacuum, n_2 is the refractive index for aluminum. When deriving formulas (4)–(6), the independence of the value n_2 from the wavelength of the radiation was assumed. With further calculations, $n_2 = 1.4$.

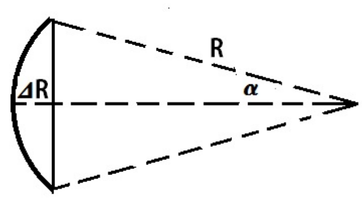

Figure 3. Spherical segment of the sail.

4 Deviation of the Tsander trajectory due to deformation of the sail surface

When moving along the Tsander trajectory, the following relation holds [14]

$$a_1 = \frac{(1-\beta)a_0}{1-2\beta}, \tag{7}$$

where a_0 is the size of the orbit's perihelion, a_1 is the length of its semimajor axis, $\beta = -\mu/M_S$, μ is the reduction due to solar radiation pressure, M_S is the mass of the Sun. In accordance with the definitions of reduction and the dimensionless parameter β, the following relationship can be written between its values for flat (β_0) and deformed (β') sails

$$\beta' = \beta_0(q_1+q_2). \tag{8}$$

Denoting by $a_{1,0}$ and a_1' the lengths of the large semi-axes of the Tsander trajectories for flat and deformed sails, respectively, from (7) we obtain

$$\delta = \frac{a_1'}{a_{1,0}} = \frac{(1-\beta')(1-2\beta)}{(1-2\beta')(1-\beta)}. \tag{9}$$

By choosing the value $\beta = 0.172$ for the Earth-Mars flight [14], from formulas (8) and (9) it is possible to determine the value of the ratio of the major semi-axes δ. This value, in turn, allows one to determine the maximum deviation of the trajectories, that is, the magnitude of the miss from a given point of flight

$$\Delta r = 2(1-\delta)a_M, \tag{10}$$

where $a_M = 1.888 \cdot 10^8 km$ is the length of the semimajor axis for the Earth-Mars flight [14]. The calculation results according to formula (10) are shown in Fig. 4.

Let us also consider the flight of a spacecraft with a solar sail between the orbits of Earth and Venus. Since a flight from Earth's orbit to Venus' orbit along the Tsander trajectory is impossible, a calculation was carried out for the return flight: from Venus' orbit to Earth's orbit. To do this, we first calculated the value $\beta \approx 0.1381$ for such a flight using the formula

$$\beta = \frac{1-c}{2-c}, \tag{11}$$

where $c = (r_E/a_V)\cdot(r_V/r_E)$, r_E is the radius of the Earth's orbit, r_V is the radius of the orbit of Venus, and a_V is the length of the semimajor axis of the Tsander Venus–Earth flight [14]. The results of calculations of the decimal logarithm of deviation for this case are shown in Fig. 5.

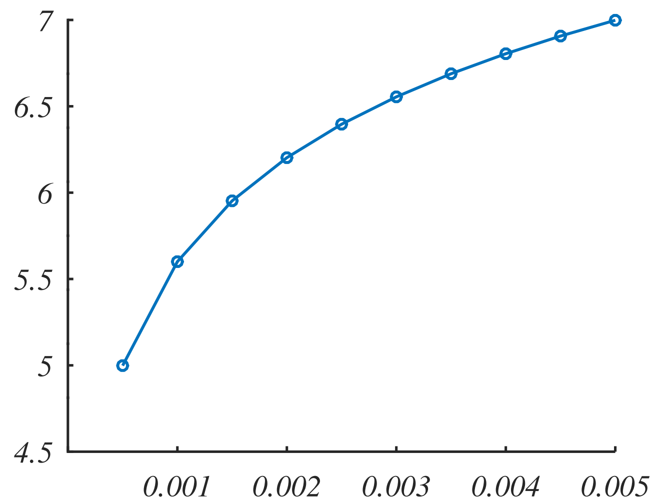

Figure 4. Earth-Mars flight: $\log(\Delta r)$ in meters versus relative deformation ε (dimensionless) of the sail.

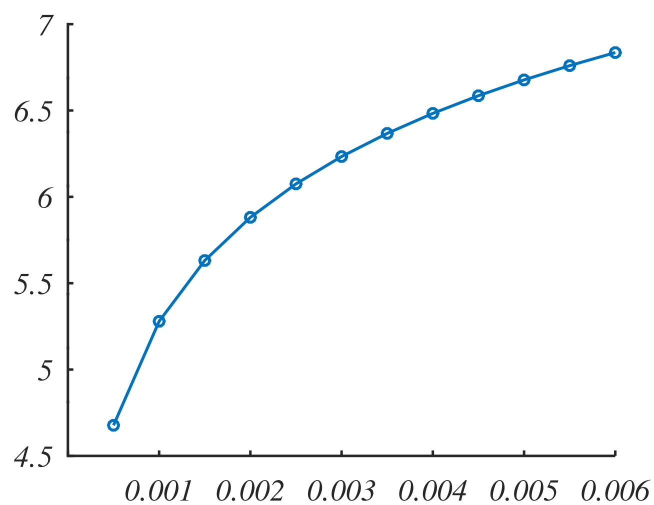

Figure 5. Venus-Earth flight: $\log(\Delta r)$ in meters versus relative deformation ε (dimensionless) of the sail.

5 Results and discussion

The calculation results show that for a solar sail with a radius of hundreds of meters required for the flight in question, even small relative deformations can lead to significant trajectory changes. So already at $\varepsilon = 0.003$ near the Earth's orbit, the deviation of the trajectories in the vicinity of Mars will exceed the radius of Mars, equal to 3394 km.

Since the relative deformations caused by the electrization of the solar sail surface will decrease as you move away from the Sun, the results shown in Fig. 4 are slightly overestimated, while in Fig. 5, on the contrary, they are slightly underestimated. However, by setting the permissible deviations, for example, near Mars Δr, we can now determine the optimal parameters of the transverse stiffeners to implement the necessary relative deviations ε in the Earth's orbit. These parameters, in turn, will allow us to calculate the value of ε at all points of the trajectory and finally calculate the exact value of the maximum deviation of the trajectories. Such a study is the objective of our future work.

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