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Application of Finite Difference Method to Study of the Phenomenon in the Theory of Thin Plates

Ć. B. Dolićanin, V. B. Nikolić, D. Ć. Dolićanin

Abstract: The term thin plate implied elastic body with cylindrical or prismatic shape of small thickens in relation to other two dimensions. The basic dependencies between geometrical and physical properties come to mostly to setting up of relations between stress and strain conditions, which has been described by differential equations, simple and partial. Methods used for solving of established equations, with respect of outline and initial conditions, may be classified in analytical and numerical. In case of complex and big construction systems subjected to the arbitrary loads, including a complex boundary conditions, solving of differential equations by analytical methods is almost impossible. Then the solution is application of numerical methods. One of the basic numerical methods is Finite Difference Method (FDM) based on replacing of differential equations with corresponding difference equations. Using of this method, the problem come to solving of system of paired algebraic equations, making the problem more easier for solving. In the end, more comments and farther directions of investigations are given.

Keywords: Numerical analysis, thin plate, finite difference method, strain

1 Introduction

Many problems in the field of deformable bodies are described by parametars of the condition of the systems which depend on coordinates, time, temperature, ect. Such systems are given in mathematical models by partial differential equations, for example, relation between stress condition and strain condition and external load in mechanics of continuum and others for solving these equations there are mainly, 1, 2, 3, 4, 5, 6, 7, 10 , two approaches: analytical and numerical. Analytical methods are of an earlier date and imply determination of mathematical functions which define solution in a closed form. The basic characteristic of numerical methods is that the fundamental equations which describe the problem, including the boundary conditions are soved in approximate numerical way. In this paper, Finite Difference Method (FDM) and the finite element method (FEM) are used for analysis of stress strain condition in the thin plates. Finite difference method (Diferencne method-FDM) is a numerical method based on mathematical discretization of differential

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Ć. B. Dolićanin, V. B. Nikolić, D. Ć. Dolićanin are with the State University of Novi Pazar, Serbia.

equations, which are the finite difference equations are translated into algebraic. The finite element method (FEM) implies discretization in physical model and the final result is obtained by solving a system of algebraic equations. FEM uses various forms of variation methods applied to discrete model. In this paper uses the finite difference method for the determination of strain state analysis of thin plates.

2 Review of finite difference method

Finite difference method is numerical methods based on mathematical discretization of the equations of boundary problems. By using this method, continuous process is studied in a finite number of sufficiently small time intervals. So it is possible, in these small intervals, the function approximated by approximate expressions. In each elementary interval is the integration, with the results of integration in the previous interval are taken as initial for the next time interval.

The function $f(x)$ is continuous in the interval AB, and some are also continuing her running $f^{(i)}(x)$ to that order, $i = 1, 2, 3, ...$ that we need. Interval AB, where A $(a, 0)$ and B$(b, 0)$, divide into equal divisions $\Delta x = h$, Fig. 1. Then the first derivative of f at x , $[1, 3, 7, 10]$, is given by the expression

$$f' = \Delta f|_x / 2h; \qquad \Delta F|_x = (f(x+h) - f(x-h))/2 \tag{1}$$

where f is the first central difference.

We can say that the first derivative is the slope of a graphical point of function $f(x)$ calculated based on the value of the function in neighboring points.

Approximate value of the first statement of $f(x)$ at intervals from x to $x+h$, ie the part of $x-h$ to x is obtained by dividing the difference with the corresponding first step interpolation h. The second derivative is

$$f'' = \frac{d^2 f}{dx^2} = \frac{\Delta^2 f}{h^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} \tag{2}$$

Analogue, can be determined, third, fourth and differences n-th row and the corresponding running.

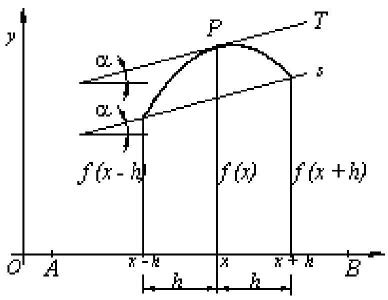


Fig. 1. The method of finite differences

If we replace the corresponding statements in the differential equations that we study, will get to a certain point (x) space (a,b) a linear equation. The total number of equations equals the number of pre-selected points "nodes"of a given space. To arrive at a solution, these equations should be added to the initial and boundary condition.

Border procedures, when the distance between the points fall to zero, gives the same threshold in the respective partial derivatives of functions given point.

In the case of two-dimensional problem that describes the partial differential equations in (x,y), .ie. in the case of $f(x,y)$ that depends on two variables require the same in most cases approximated by applying the finite difference method, to solve the problem using a computer, $[1,2,3,7,9,10]$. To calculate the partial statement, suppose that the known values of the function fu network nodal points of the budget, etc. Fig. 2.

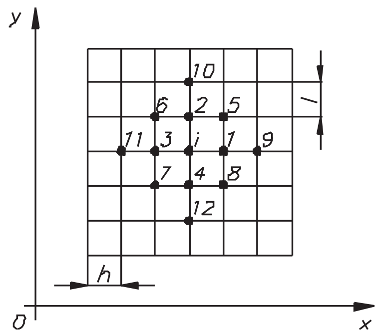


Fig. 2. Networks for solving two-dimensional problem

In the point $i(x,y)$ observed rectangular grid with power ranges $\Delta x = h$ (on the x axis) and $\Delta y = l$ (on the y axis), the first partial derivatives, analogous to the previous procedure, are

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\Delta_x f}{2h} = \frac{f_1 - f_3}{2h} = \frac{1}{2h}(f(x+h,y) - f(x-h,y)) \\ \frac{\partial f}{\partial y} &= \frac{\Delta_y f}{2l} = \frac{1}{2l}(f(x,y+l) - f(x,y-l)) \end{aligned} \tag{3}$$

where are $\Delta_x f$ and $\Delta_y f$ the first partial derivatives for x and y directions. Analogously, the partial derivatives of second, third and fourth order we have

$$\begin{aligned} \frac{\partial^2}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\Delta_x^2 f}{h^2} = \frac{f_1 - 2f_i + f_3}{h^2} = \frac{1}{h^2}\left(f(x+h,y) - 2f(x,y) + f(x-h,y)\right); \\ \frac{\partial^2}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\Delta_y^2 f}{l^2} = \frac{f_2 - 2f_i + f_4}{l^2} = \frac{1}{l^2}\left(f(x,y+l) - 2f(x,y) + f(x,y-l)\right); \\ \frac{\partial^3 f}{\partial x^3} &= \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\Delta_x^3 f}{2h^3} = \frac{1}{2h^3}\left(f(x+2h,y) - 2f(x+h,y) + 2f(x-h,y) - f(x-2h,y)\right); \\ \frac{\partial^3 f}{\partial y^3} &= \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right) = \frac{\Delta_y^3 f}{2l^3} = \frac{1}{2l^3}\left(f(x,y+2l) - 2f(x,y+l) + 2f(x,y-l) - f(x,y-2\,l)\right); \end{aligned} \tag{4}$$

$$\begin{aligned}
\frac{\partial^4 f}{\partial x^4} &= \frac{\partial}{\partial x}\left(\frac{\partial^3 f}{\partial x^3}\right) = \frac{\Delta_x^4 f}{h^4} = \frac{f_9 - 4f_1 + 6f_i - 4f_3 + f_{11}}{h^4} = \\
&= \frac{1}{h^4}\left(f(x+2h,y) - 4f(x+h,y) + 6f(x,y) - 4f(x-h,y) + f(x-2h,y)\right) \\
\frac{\partial^4 f}{\partial y^4} &= \frac{\partial}{\partial y}\left(\frac{\partial^3 f}{\partial y^4}\right) = \frac{\Delta_y^4 f}{l^4} = \frac{f_{10} - 4f_2 + 6f_i - 4f_4 + f_{12}}{l^4} = \\
&= \frac{1}{l^4}\left(f(x,y+2l) - 4f(x,y+l) + 6f(x,y) - 4f(x,y-l) + f(x,y-2l)\right)
\end{aligned}$$

In a similar way can be derived in terms

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{1}{2l}\left(\frac{\partial f}{\partial x_{)2}} - \frac{\partial f}{\partial x_{)4}}\right) \tag{5}$$

Substituting these expressions into the corresponding partial differential equations of the problem at the same transform the finite difference equation or difference equation . Also, the total number of equations equals the number of pre-selected points or nodes of a given space on the x and y axes.

3 Examples of solution for strain in thin plates

Under a thin plate in the theory of elasticity of the elastic body includes a cylindrical or prismatic shape thickness h in relation to the other two dimensions, a and b, [3,6,7,10] . Plate thickness should be less than 1/10 of other dimensions. Medium flat panel takes up the $0xy$ plane, and axis $0z$ is directed downward in the direction of the load which the unit of surface is p, $\mathrm{N/mm^2}$, Fig. 3 and Fig. 4.

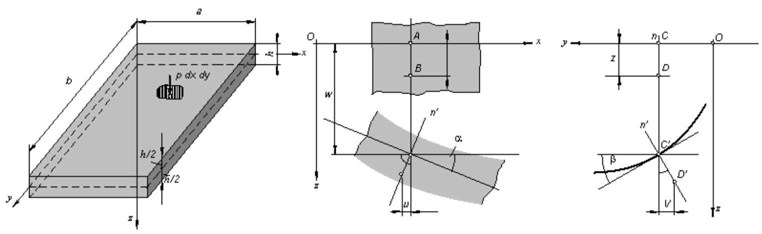


Fig. 3. Bending of thin plates.

Due to the small size h plate can be considered as points of attack power loads in the very high level. Due to loads p perform deformation plate. It is assumed that the deflections are very small, so that the upper limit of the size of the strain should not exceed the size of $h/5$. Some high points of the plane deflections $w = w_0 = w(x,y)$ and functions are just the coordinates x and y. The mean plane by deformation exceeds the elastic surface.

In the plate theory is introduced hypotheses linear element as follows: normal to the middle plane remains the administrative and the elastic surface, which means that the linear elements remain unchanged by deformation length. The plate theory assumes that the $\sigma_z =$

0, and the mean plane of normal stress and strain in the flat, zero, neutral surface (plane). It is further assumed that no body forces act.

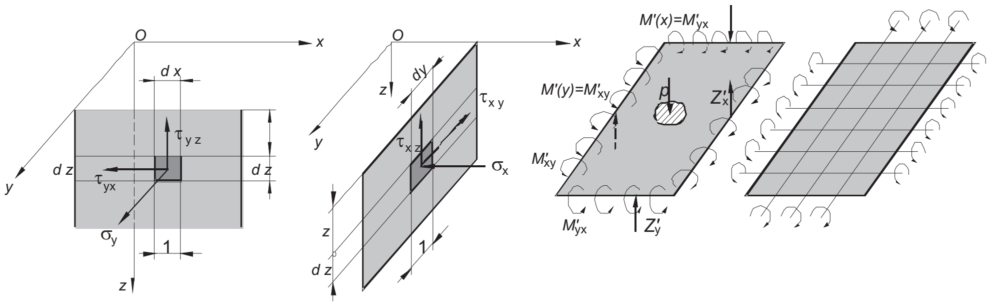


Fig. 4. Force reduction.

To perform differential equations of equilibrium, ie. differential equation of the elastic surface plate, it is enough to observe the element $dxdydz$ separated from the plate. In this isolated element acting bending moments M_x and M'_x, or M_y and M'_y , moments torsion T_{txy} and T'_{txy} or, T_{tyx} and T'_{tyx}, transversal forces Q_x and Q'_x, Q_y and Q'_y and the given external load, etc.

By placing the link between stress and moments, stress and strain, we get the equations of equilibrium in the form of

$$\Delta\Delta\omega = \frac{\partial^4 \omega}{\partial x^4} + 2\frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{p}{D} \tag{6}$$

where is $D = \frac{Eh^3}{12(1-\mu^3)}$ N/mm^2 plate bending stiffness or cylindrical plate bending stiffness (flexural rigidity of the plate). It is analogous savojnoj stiffness beams. Finite difference form, of difference equation (6), $[3,6,7,10]$, according to the scheme on Fig. 2, is:

$$\Delta\Delta\omega = 20\omega_i - 8(\omega_1 + \omega_2 + \omega_3 + \omega_4) + 2(\omega_5 + \omega_6 + \omega_7 + \omega_8) + (\omega_9 + \omega_{10} + \omega_{11} + \omega_{12}) = \frac{p}{D}h^4 \tag{7}$$

When the known dimensions and known when the load $p(x,y)$ rectangular plate, it should solve the partial elastic surface equation (6), and the system of difference equations (7). The solution is required to satisfy boundary (boundary) conditions on the side panels parallel basis $0x$ and $0y$ axes. There are four types of suspension lateral basis (side) panels: the flex side (base), simply supported (supported) pages, free pages and elastically supported and elastically flex side $[3,6,7,10]$.

In the paper given, apply finite difference method, three examples of solutions for the deflection plate under a given load .

Example 1: Rectangular plate flex sides AB and CD, simply supported by the ad and free side BC is loaded as in Figure 5. It is known: $F = 1kN$; $a = 4m$; $b = 6m$; $\sigma = 0.1m$; $\mu = 0,3$; $E = 2.110^5 MPa$.

By Finite difference method we will be certain values of strain at nodal points of the adopted network.

For the flex side of the deflection equals zero, the free, supported by foreign side of the deflections and bending moments are equal to zero and on the free side, there is no bending or twisting, and transversal forces do not act.

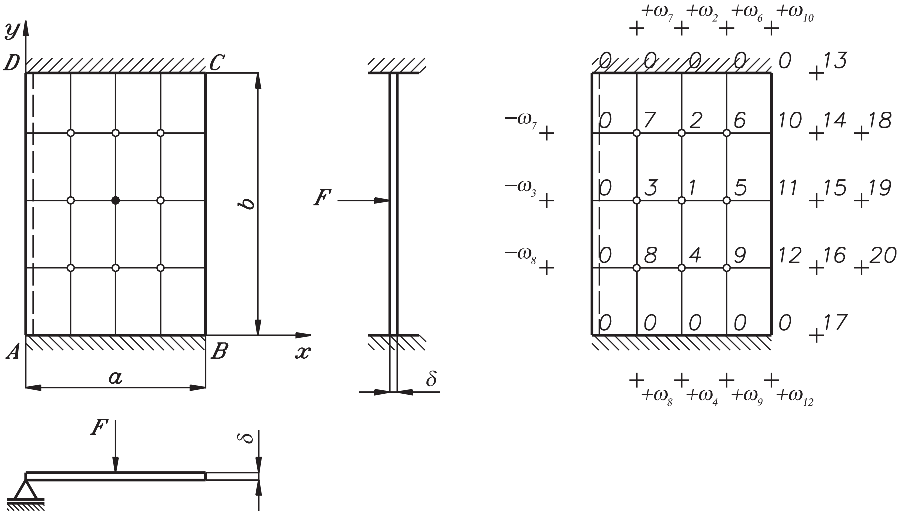


Fig. 5. A rectangular plate is loaded in the concentrated force of gravity .

Deflection equation is given by (6) and Diferencne form expression (7). If you introduce tags

$$l = b/4 = 6/4 = 1.5m; \qquad h = a/4 = 5/5 = 1m;$$

$$\alpha = \frac{l^2}{h^2} = \frac{9}{4} = 2.25; \qquad p_i = \frac{F}{h \cdot l} = \frac{1000}{1 \cdot 1.5} = \frac{2000}{3} = 666.667\frac{N}{m^2}$$

$$D = \frac{E\delta^3}{12(1-\mu^2)} = \frac{2.1 \cdot 10^{11} \cdot 0.1^3}{12(1-0.3^2)} = 19.23077 \cdot 10^6 Nm$$

obtained by members of equations (7) in the form of

$$6\alpha^2 + 8\alpha + 6 = 54.375; \quad 4\alpha(\alpha+1) = 29.25; \quad 4(\alpha+1) = 13; \; 2\alpha = 4.5; \; \alpha^2 = 5.0625$$

Concentrated force F, pressure is defined as $p = F/hl$ and a force that increases the demand node $F = phl$.

Taking into account the symmetry of the plate relative to the y-axis and the boundary conditions of reliance, there is a system of algebraic equations (according to the scheme in Fig. 2):

node 1:

$$54.375\omega_1 - 29.25(\omega_3 + \omega_5) - 13(\omega_2 + \omega_4) + 4.5(\omega_6 + \omega_7 + \omega_8 + \omega_9) +$$
$$+ \quad 5.0625(0 + \omega_{11}) + 0 + 0 = 175.499$$

node 3:

$$\begin{aligned} & 54.375\omega_3 - 29.25\,(0+\omega_1) - 13\,(\omega_7+\omega_8) + 4.5\,(0+\omega_4+\omega_2+0) + \\ + \quad & 5.0625\,(\omega_5-\omega_3) + 0 + 0 = 0 \end{aligned}$$

node 5:

$$\begin{aligned} & 54.375\omega_5 - 29.25\,(\omega_1+\omega_{11}) - 13\,(\omega_6+\omega_9) + 4.5\,(\omega_4+\omega_{12}+\omega_{10}+\omega_2) + \\ + \quad & 5.0625\,(\omega_3+\omega_{15}) + 0 + 0 = 0 \end{aligned}$$

node 11:

$$\begin{aligned} & 54.375\omega_{11} - 29.25\,(\omega_5+\omega_{15}) - 13\,(\omega_{10}+\omega_{12}) + 4.5\,(\omega_9+\omega_{16}+\omega_{14}+\omega_6) + \\ + \quad & 5.0625\,(\omega_1+\omega_{19}) + 0 + 0 = 0 \end{aligned}$$

node 7:

$$\begin{aligned} & 54.375\omega_7 - 29.25\,(0+\omega_2) - 13\,(0+\omega_3) + 4.5\,(0+\omega_1+0+0) + \\ + \quad & 5.0625\,(\omega_6-\omega_7) + \omega_7 + \omega_8 = 0 \end{aligned}$$

node 2:

$$\begin{aligned} & 54.375\omega_2 - 29.25\,(\omega_7+\omega_6) - 13\,(\omega_1+0) + 4.5\,(\omega_3+\omega_5+0+0) + \\ + \quad & 5.0625\,(0+\omega_{10}) + \omega_4 + \omega_2 = 0 \end{aligned}$$

node 6:

$$\begin{aligned} & 54.375\omega_6 - 29.25\,(\omega_2+\omega_{10}) - 13\,(\omega_5+0) + 4.5\,(\omega_1+\omega_{11}+0+0) + \\ + \quad & 5.0625\,(\omega_7+\omega_{14}) + \omega_9 + \omega_6 = 0 \end{aligned}$$

node 10:

$$\begin{aligned} & 54.375\omega_{10} - 29.25\,(\omega_6+\omega_{14}) - 13\,(\omega_{11}+0) + 4.5\,(\omega_5+\omega_{15}+\omega_{13}+0) + \\ + \quad & 5.0625\,(\omega_2+\omega_{18}) + \omega_{10} + \omega_{12} = 0 \end{aligned}$$

Helpfully sizes:

$$\tfrac{\mu}{\alpha} = \tfrac{0.3}{2.25} = 0.1333 \qquad 4\left(1+\tfrac{2-\mu}{\alpha}\right) = 7.022$$

$$2\left(1+\tfrac{\mu}{\alpha}\right) = 2.267 \qquad 4\left(\tfrac{1}{\alpha}-\tfrac{2\mu-\mu^2}{\alpha^2}\right) = 2.18074$$

$$4\left(1+\tfrac{2}{\alpha}+\tfrac{3}{2}\tfrac{2\mu-\mu^2}{\alpha^2}\right) = 8.16 \qquad 2\frac{2-\mu}{\alpha} = 1.5111$$

$$\frac{2\mu-\mu^2}{\alpha^2} = 0.10074$$

1 line: $\omega_{13} = 2.267 \cdot 0 - 0 - 0.1333\,(\omega_{10} + \omega_{10})$
$\omega_{14} = 2.267 \cdot \omega_{10} - \omega_6 - 0.1333\,(\omega_{11} + 0)$;
$\omega_{15} = 2.267 \cdot \omega_{11} - \omega_5 - 0.1333\,(\omega_{10} + \omega_{12})$

2 line: $\omega_{18} = 8.16 \cdot \omega_{10} - 7.022\omega_6 - 2.18074\,(\omega_{11} + 0) + 1.511\,(\omega_5 + 0) +$
$+ \omega_2 + 0.10074\,(\omega_{10} + \omega_{12})$
$\omega_{19} = 8.16 \cdot \omega_{11} - 7.022\omega_5 - 2.18074\,(\omega_{10} + \omega_{12}) + 1.511\,(\omega_9 + \omega_6) +$
$+ \omega_1 + 0.10074\,(0 + 0)$

After ordering we get a system of equations whose solving in Mathcad using the Find function Given the values of deflection in the requested budget network nodes:

1 line

given

$$54.375\omega_1 - 26\omega_2 - 29.25\omega_3 - 29.25\omega_5 + 9\omega_6 + 9\omega_7 + 0\omega_{10} + 5.0625\omega_{11} = 175.499\cdot 10^{-6}$$

$$-29.25\omega_1 + 9\omega_2 + 49.3125\omega_3 + 5.0625\omega_5 + 0\omega_6 - 26\omega_7 + 0\omega_{10} + 0\omega_{11} = 0$$

$$-29.25\omega_1 + 9\omega_2 + 5.0625\omega_3 - 49.3125\omega_5 - 26\omega_6 - 0\omega_7 + 7.6503\omega_{10} - 17.773\omega_{11} = 0$$

$$10.125\omega_1 + 0\omega_2 + 0\omega_3 - 35.5489\omega_5 + 15.299\omega_6 + 0\omega_7 - 19.8799\omega_{10} - 28.1755\omega_{11} = 0$$

$$4.5\omega_1 - 29.25\omega_2 - 13\omega_3 + 0\omega_5 + 5.0625\omega_6 + 51.31252\omega_7 + 0\omega_{10} + 0\omega_{11} = 0$$

$$-13\omega_1 + 56.375\omega_2 + 4.5\omega_3 + 4.5\omega_5 - 29.25\omega_6 - 29.25\omega_7 + 5.0625\omega_{10} + 0\omega_{11} = 0$$

$$4.5\omega_1 - 29.25\omega_2 + 0\omega_3 - 13\omega_5 + 51.3125\omega_6 + 5.0625\omega_7 - 17.7733\omega_{10} + 3.825\omega_{11} = 0$$

$$0\omega_1 + 10.125\omega_2 + 0\omega_3 + 7.6499\omega_5 - 35.5489\omega_6 + 0\omega_7 + 29.9958\omega_{10} - 11.07297\omega_{11} = 0$$

$$\operatorname{find}(\omega_1, \omega_2, \omega_3, \omega_5, \omega_6, \omega_7, \omega_{10}, \omega_{11}) \rightarrow \begin{pmatrix} 3.3782068488235194989 \cdot 10^{-6} \\ 1.0332885227400467117 \cdot 10^{-6} \\ 2.5752521219211802998 \cdot 10^{-6} \\ -2.4220254509186598866 \cdot 10^{-6} \\ -2.5056705797307480518 \cdot 10^{-7} \\ 9.6991010744365241995 \cdot 10^{-7} \\ 1.1884074553557873549 \cdot 10^{-6} \\ 3.2952685171458499366 \cdot 10^{-6} \end{pmatrix}$$

The values of deflection can be considered relevant, because this method, previously, tested on many examples.

Example 2. Rectangular plate flex sides AB and CD, simply supported sides AD and BC is loaded as in Figure 6 continuous load q. It is known: $E = 2.1 \cdot 10^5$ MPa, $\mu = 0.3$; $\delta = 0.1$m, q = 2kN/m^2, a = 4m, b = 6m.

With Finite difference method will be determine the values of deflection at nodal points of the adopted network.

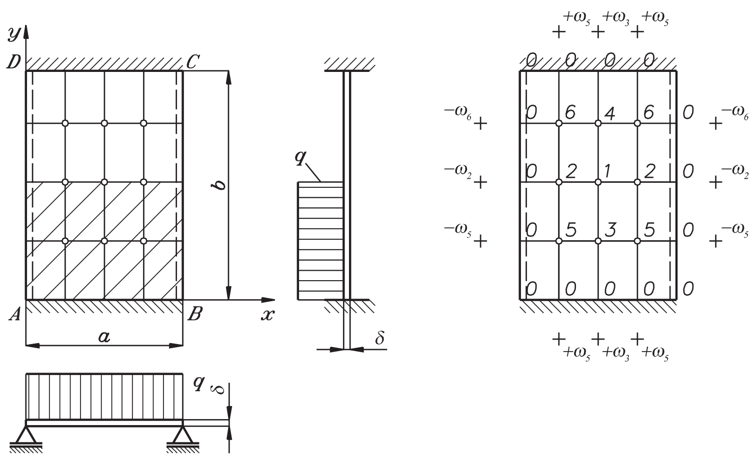


Fig. 6. A rectangular plate partially loaded continuous load.

Note: Analogous to the previous example, the case when the surface of the panel operates continuously load q, a node that is located inside the area is burdened by power qlh, and for node 0, Figure 7 a) p = q. In other words, node 0 receives a continuous burden on themselves by acting on the surface of the rectangle width height h and l, where it is located in the center of the rectangle.In contrast nodes 1, 2, 3 and 4 received half so defined load, Figure 7 b), and nodes 5, 6, 7 and 8, only one quarter, Figure 7 c). So the pressure values

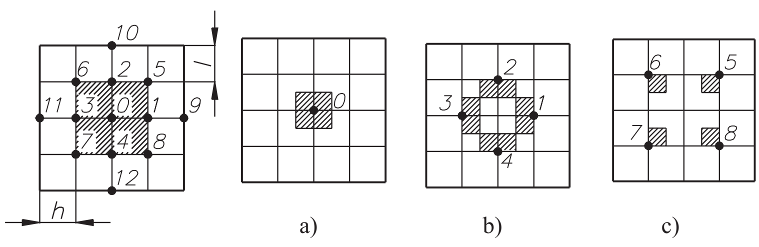


Fig. 7. Scheme for determining force in the nodal point

in the corresponding nodes will be:

$$p_0 = q,\ p_1 = p_2 = p_3 = p_4 = \frac{q}{2},\ p_5 = p_6 = p_7 = p_8 = \frac{q}{4}$$

$$\frac{\partial^4 \omega}{\partial x^4}+2\frac{\partial^4 \omega}{\partial x^2 \partial y^2}+\frac{\partial^4 \omega}{\partial y^4}=\frac{p}{D}$$

Applying MKR is obtained by the equilibrium equations of the form:

$$\begin{aligned}
& \left(6\alpha^2+8\alpha+6\right)\omega_i-4\alpha(\alpha+1)(\omega_1+\omega_3)-4(\alpha+1)(\omega_2+\omega_4)+ \\
+ \quad & 2\alpha(\omega_5+\omega_6+\omega_7+\omega_8)+\alpha^2(\omega_9+\omega_{11})+\omega_{10}+\omega_{12}=\frac{p_i l^4}{D}
\end{aligned}$$

$$l=b/4=6/4=1.5m; \qquad h=a/4=4/4=1m$$

$$\begin{aligned}
\alpha &= \frac{l^2}{h^2}=\frac{9}{4}=2.25 \qquad p_1=p_2=\frac{q}{2}=1\frac{kN}{m^2} \qquad p_3=p_5=q=2\frac{N}{m^2} \\
D &= \frac{E\delta^3}{12(1-\mu^2)}=\frac{2.1\cdot 10^{11}\cdot 0.1^3}{12(1-0.3^2)}=19.23077\cdot 10^6 Nm
\end{aligned}$$

$$6\alpha^2+8\alpha+6=54.375;\ 4\alpha(\alpha+1)=29.25;\ 4(\alpha+1)=13;\ 2\alpha=4.5\ \alpha^2=5.0625$$

node 1: $54.375\omega_1-29.25(\omega_2+\omega_2)-13(\omega_3+\omega_4)+4.5(\omega_5+\omega_5+\omega_6+\omega_6)+$
$+5.0625(0+0)+0+0=0.26325\cdot 10^{-3}$

node 2: $54.375\omega_2-29.25(0+\omega_1)-13(\omega_5+\omega_6)+4.5(0+0+\omega_3+\omega_4)+$
$+5.0625(\omega_2-\omega_2)+0+0=0.26325\cdot 10^{-3}$

node 3: $54.375\omega_3-29.25(\omega_5+\omega_5)-13(0+\omega_1)+4.5(0+0+\omega_2+\omega_2)+$
$+5.0625(0+0)+\omega_3+\omega_4=0.5625\cdot 10^{-3}$

node 4: $54.375\omega_4-29.25(\omega_6+\omega_6)-13(0+\omega_1)+4.5(0+0+\omega_2+\omega_2)+$
$+5.0625(0+0)+\omega_4+\omega_3=0$

node 5: $54.375\omega_5-29.25(0+\omega_3)-13(0+\omega_2)+4.5(0+0+0+\omega_1)+$
$+5.0625(\omega_5-\omega_5)+\omega_5+\omega_6=0.5625\cdot 10^{-3}$

node 6: $54.375\omega_6-29.25(0+\omega_4)-13(0+\omega_2)+4.5(0+0+0+\omega_1)+$
$+5.0625(-\omega_6+\omega_6)+\omega_6+\omega_5=0$

given

$$54.375\omega_1 - 58.5\omega_2 - 13\omega_3 - 13\omega_4 + 9\omega_5 + 9\omega_6 = 0.26325 \cdot 10^{-3}$$

$$-29.25\omega_1 + 54.375\omega_2 + 4.5\omega_3 + 4.5\omega_4 - 13\omega_5 - 13\omega_6 = 0.26325 \cdot 10^{-3}$$

$$-13\omega_1 + 9\omega_2 + 55.375\omega_3 + \omega_4 - 58.5\omega_5 + 0\omega_6 = 0.5625 \cdot 10^{-3}$$

$$-13\omega_1 + 9\omega_2 + \omega_3 + 55.375\omega_4 + 0\omega_5 - 58.5\omega_6 = 0$$

$$4.5\omega_1 - 13\omega_2 - 29.25\omega_3 + 0\omega_4 + 55.375\omega_5 + \omega_6 = 0.5265 \cdot 10^{-3}$$

$$4.5\omega_1 - 13\omega_2 + 0\omega_3 - 29.25\omega_4 + \omega_5 + 55.375\omega_6 = 0$$

$$\mathrm{find}(\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6) \rightarrow \begin{pmatrix} 8.4135943688372158218 * 10^{-5} \\ 6.0658239523563842244 * 10^{-5} \\ 8.3348495453246540610 * 10^{-5} \\ 3.4062732779655031070 * 10^{-5} \\ 6.0498163354981516936 * 10^{-5} \\ 2.4303063434015049735 * 10^{-5} \end{pmatrix}$$

Example 3. Rectangular plate simply supported on all its four sides is loaded as in Figure 8 triangular continuous load q. Apply Finite differences Method to find the values of deflection at nodal points of the adopted network. It is known: q= 4*kN/m*^{2}; $a = 6$m; $b = 8$m; δ= 0.1*m;* μ= 0,3; E= $2{\cdot}10^5$ *MPa.* The governing equations

$$\frac{\partial^4\omega}{\partial x^4} + 2\frac{\partial^4\omega}{\partial x^2 \partial y^2} + \frac{\partial^4\omega}{\partial y^4} = \frac{p}{D}$$

$$\begin{aligned} &\left(6\alpha^2 + 8\alpha + 6\right)\omega_i - 4\alpha(\alpha+1)(\omega_1+\omega_3) - 4(\alpha+1)(\omega_2+\omega_4) + \\ + \quad & 2\alpha(\omega_5+\omega_6+\omega_7+\omega_8) + \alpha^2(\omega_9+\omega_{11}) + \omega_{10} + \omega_{12} = \frac{p_i l^4}{D} \end{aligned}$$

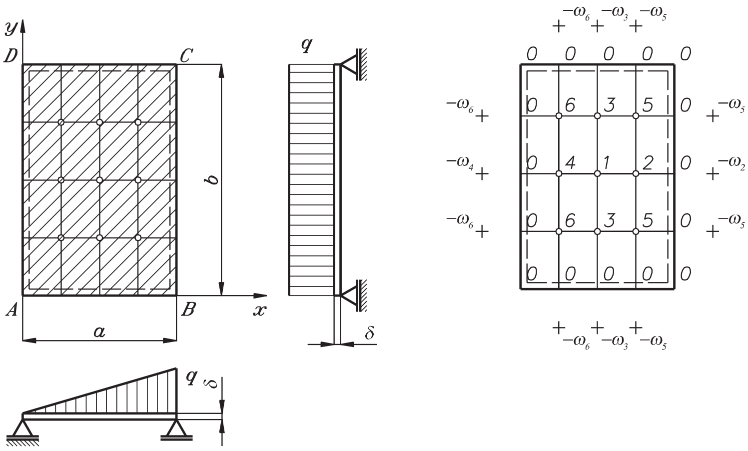


Fig. 8. A rectangular plate with a continuous load.

Under the given conditions, we get

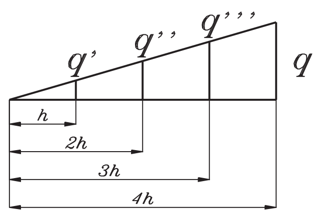
$$
\begin{aligned}
l &= b/4 = 8/4 = 2m; \qquad h = a/4 = 6/4 = 3/2m;\\
\alpha &= \frac{l^2}{h^2} = \frac{16}{9}\\
D &= \frac{E\delta^3}{12\,(1-\mu^2)} = \frac{2\cdot 10^{11}\cdot 0.1^3}{12\,(1-0.3^2)} = 18.315\cdot 10^6 Nm
\end{aligned}
$$

Pressure values in the corresponding nodes:

$q : q' = 4h : h \Rightarrow q' = q/4 \Rightarrow q_4 = q_6 = q/4$

$q : q'' = 4h : 2h \Rightarrow q'' = q/2 \Rightarrow q_1 = q_3 = q/2$

$q : q''' = 4h : 3h \Rightarrow q''' = 3q/4 \Rightarrow q_2 = q_5 = 3q/4$



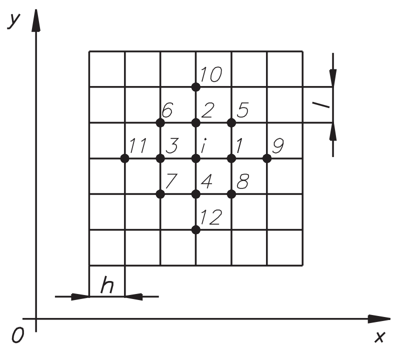
$$6\alpha^2 + 8\alpha + 6 = \frac{3174}{81}$$

$$4\alpha(\alpha+1) = \frac{1600}{81}$$

$$4(\alpha+1) = \frac{100}{9}$$

$$2\alpha = \frac{32}{9}$$

$$\alpha^2 = \frac{256}{81}$$



Starting from the equation in the form (7):

$$\left(6\alpha^2+8\alpha+6\right)\omega_i-4\alpha\left(\alpha+1\right)\left(\omega_1+\omega_3\right)-4\left(\alpha+1\right)\left(\omega_2+\omega_4\right)+$$
$$+\quad 2\alpha\left(\omega_5+\omega_6+\omega_7+\omega_8\right)+\alpha^2\left(\omega_9+\omega_{11}\right)+\omega_{10}+\omega_{12}=\frac{p_i l^4}{D}$$

Can be written the equation for individual nodes:

node 1: $3174\omega_1-1600\left(\omega_2+\omega_4\right)-900\left(\omega_3+\omega_3\right)+288\left(\omega_5+\omega_5+\omega_6+\omega_6\right)+$
$+256\left(0+0\right)+0+0=81\frac{p_1 l^4}{D}$

node 2: $3174\omega_2-1600\left(0+\omega_4\right)-900\left(\omega_5+\omega_5\right)+288\left(0+\omega_3+\omega_3+0\right)+$
$+256\left(-\omega_2+\omega_4\right)+0+0=81\frac{p_2 l^4}{D}$

node 3: $3174\omega_3-1600\left(\omega_5+\omega_6\right)-900\left(0+\omega_1\right)+288\left(\omega_5+0+\omega_6+\omega_1\right)+$
$+256\left(0+0\right)-\omega_3+\omega_3=81\frac{p_3 l^4}{D}$

node 4: $3174\omega_4-1600\left(\omega_1+0\right)-900\left(\omega_6+\omega_6\right)+288\left(\omega_3+0+0+\omega_3\right)+$
$+256\left(\omega_2-\omega_4\right)+0+0=81\frac{p_4 l^4}{D}$

node 5: $3174\omega_5-1600\left(0+\omega_3\right)-900\left(\omega_2+\omega_0\right)+288\left(0+\omega_1+0+0\right)+$
$+256\left(-\omega_5+\omega_6\right)+\omega_5-\omega_5=81\frac{p_5 l^4}{D}$

node 6: $3174\omega_6-1600\left(\omega_3+0\right)-900\left(\omega_4+\omega_0\right)+288\left(\omega_1+0+0+0\right)+$
$+256\left(\omega_5-\omega_6\right)+\omega_6-\omega_6=81\frac{p_6 l^4}{D}$

After ordering we get a system of equations whose solving in Mathcad using the Find function Given the values of the stress functions in the requested budget network nodes:

given

$$3174\omega_1-1600\omega_2-1800\omega_3-1600\omega_4+576\omega_5+576\omega_6=0.14152$$

$$-1600\omega_1+2918\omega_2+576\omega_3+256\omega_4-1800\omega_5=0.21228$$

$$-900\omega_1+288\omega_2+3174\omega_3+288\omega_4-1600\omega_5-1600\omega_6=0.14152$$

$$-1600\omega_1+256\omega_2+576\omega_3+2918\omega_4-1800\omega_6=0.07076$$

$$288\omega_1-900\omega_2-1600\omega_3+2918\omega_5+256\omega_6=0.21228$$

$$288\omega_1-1600\omega_3-900\omega_4+256\omega_5+2918\omega_6=0.07076$$

$$\text{find}\left(\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6\right)\rightarrow\begin{pmatrix} 9.3047341809663193522\cdot 10^{-4} \\ 7.2847116725436574292\cdot 10^{-4} \\ 6.8788099127496445163\cdot 10^{-4} \\ 6.1295085019570535381\cdot 10^{-4} \\ 5.4320903651006013122\cdot 10^{-4} \\ 4.5098954539330793355\cdot 10^{-4} \end{pmatrix}$$

4 Conclusion

Summarizing the results presented in this paper, the conclusion that the finite difference metod (MCD) can effectively solve the problems of bending of thin plates and bulges and at various loads. The developed methodology makes it much easier to find solutions for the plate deflection, moments, stresses, strains, etc., which provides a significant advantage over conventional analytical methods. Further study may include temperature strains, dynamic load and the like By using this method can be dealt with more complex problems, [5, 8, 11, 12], what will be the subject of the future,

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